10. P. V. Trusov, "Construction of a model of loading process and method of correcting analysis in the study of large plastic deformations," Perm' Polytechnical Institute, Perm' (1984), VINITI, No. 5939-84, Moscow (1984).
11. P. V. Trusov, V. V. Mulyukov, and V. D. Oniskiv, "Corotational derivatives and defining relations in the theory of large deformations," Perm' Polytechnical Institute, Perm' (1985), VINITI, No. 8020-85 (1985).
12. V. Lubarda, "The elastic-plastic constitutive relation," Teor. Primen. Mekh., 7, 101 (1981).
13. F. Sidoroff, "Incremental constitutive equation for large strain elastoplasticity," Int. J. Eng. Sci., 20, No. 2 (1982).
14. V. S. Lenskii, "Current questions and problems in plasticity in theoretical and applied aspects," in: Elasticity and Inelasticity [in Russian], Moscow State University Press, Moscow (1978), No. 5.
15. A. A. Il'yushin, Plasticity. Foundations of the General Mathematical Theory [in Russian], USSR Academy of Sciences Press, Moscow (1963).

GENERAL ENERGY RELATIONS FOR RAIL GUNS

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The question of energy relations is one of the most important questions in the description of the operation of rail guns and the analysis of their potential possibilities. For limited energy storage in the supply source, the efficiency of the conversion of electromagnetic energy into kinetic energy of the accelerated body essentially determines the possible scale of the experiment. There are a large number of papers devoted to this problem (see, e.g., [1-9]). In the analysis of the energy conversion efficiency there arises the question of the optimal choice of the parameters of the system under study, ensuring the most efficient conversion of energy stored in the source into the kinetic energy of the accelerated body: the energy of the source, the inductance and active resistance of the circuit, the inductance per unit length of the accelerator, the mass of the body, and other parameters characterizing only the given type of source. Since the number of determining parameters is small, the main method reduced to the numerical solution of the equations determining the acceleration process together with analytical study of the asymptotic states. This approach is developed in particular in the works cited above. Numerical solutions permit determining the optimal parameters of the system for a given energy source, but do not fully reveal the general energy relations, characteristic of the rail gun method for accelerating solids.

The main analytical solutions, which permit evaluating, under a number of assumptions, the velocity and efficiency with which the energy stored in the source is converted into kinetic energy of the body, were obtained in [1-3, 5, 6]. Without examining in detail the assumptions made by the authors, we point out only that an assumption common to all of them is that the active resistance of the circuit equals zero. As a consequence of this assumption it was concluded that with a correct choice of the parameters of the accelerator and source the conversion efficiency can be close to unity. At the same time the results of a numerical solution of the equations and qualitative considerations regarding the ohmic losses show that the effect of the active resistance of the circuit on the velocity of the body and the efficiency can be very significant [5-8]. In [7, 8] it is pointed out that ohmic losses in real accelerators can substantially limit the possibility of rail guns.

The most complete analysis of the energy distribution in a circuit consisting of a capacitor bank and a rail gun has apparently been carried out in [9] for the experiments described in the literature. For the acceleration of particles with a mass of 0.3 g a maximum velocity of 3.3 km/sec with an energy conversion efficiency of ~1% was achieved. The ratio of the kinetic energy of the body to the ohmic energy dissipated in the plasma bridge, accelerating a dielectric body, equalled approximately 13%. Data which would permit evaluating the ratio

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TABLE 1

v_*, km/sec	R_0, Ω	λ, GN/m	v_*, km/sec	R_0, Ω	λ, GN/m	v_*, km/sec	R_0, Ω	λ, GN/m
10	10^{-3}	$4\cdot10^{-7}$	20	10^{-3}	$2\cdot10^{-7}$	40	10^{-3}	10^{-7}
30	$3\cdot10^{-3}$	$4\cdot10^{-7}$	60	$3\cdot10^{-3}$	$2\cdot10^{-7}$	120	$3\cdot10^{-3}$	10^{-7}
50	$5\cdot10^{-3}$	$4\cdot10^{-7}$	100	$5\cdot10^{-3}$	$2\cdot10^{-7}$	200	$5\cdot10^{-3}$	10^{-7}

of the kinetic energy of the body to the total ohmic losses in the circuit were not presented, but it is obvious that it is small and falls between the two values indicated.

Thus the results of experiments and numerical modeling show that the effect of the active resistance of the circuit on the efficiency of the accelerator operation can be very significant and it must be taken into account when the maximum possibilities of rail guns are evaluated.

This paper is concerned with the study of the dependence of the energy characteristics of rail guns on the basic parameters of the circuit, including the active resistance. The ratio of the kinetic energy of the body and the ohmic losses in the circuit is studied.

1. We represent the equations of motion and energy balance in the rail gun in the form

$$m\frac{dv}{dt} = \frac{\lambda}{2}I^2; \tag{1.1}$$

$$W_0 = E_k(t) + E_R(t) + W(t), \tag{1.2}$$

where m and v are the mass and velocity of the body; $\lambda = dL/dx$ is the inductance per unit length of the rail gun; I is the current; W_0 is the initial energy of the source; $E_k(t) = mv^2/2$ is the kinetic energy of the accelerated body; $E_R(t) = \int_0^t RI^2dt$ are the ohmic losses in circuit; R is the resistance of the circuit; and W(t) is the instantaneous value of the electric and magnetic energy in the circuit.

We rewrite the expression (1.2) as follows

$$\eta_k(t) + \eta_R(t) = 1 - \varepsilon(t). \tag{1.3}$$

Here $\eta_k(t)$ is the efficiency of conversion of the energy W_0 into kinetic energy of the body, $\eta_R(t)$ characterizes the relative fraction of ohmic losses; and, $\varepsilon(t) = W(t)/W_0$.

If it is assumed that the active resistance of the circuit remains constant during the operation of the accelerator, i.e., $R = R_0 = \text{const}$ and $v(0) = (0)$, then combining the expression for $E_R(t)$ with Eq. (1.1) we obtain

$$E_R = \frac{2mR_0v}{\lambda}; \tag{1.4}$$

$$\frac{E_k}{E_R} = \frac{v}{v_*}; \tag{1.5}$$

$$v_* = \frac{4R_0}{\lambda} \tag{1.6}$$

(v_* is the characteristic velocity). For $v < v_*$, $E_k < E_R$ and for $v > v_*$, $E_k > E_R$. As an illustration Table 1 gives the values of v_* calculated for $\lambda = 4\cdot10^{-7}$, $2\cdot10^{-7}$ and 10^{-7} H/m, characteristic for rail guns with square, rectangular, and coaxial cross sections. It should be noted that the ratio of the kinetic energy of the body and the ohmic losses in the circuit (1.5) is independent of the shape of the current pulse.

Combining the expressions (1.3) and (1.5) we can obtain η_k, η_R, and v as functions of v_*:

$$\eta_k = \frac{\frac{v}{v_*}}{1+\frac{v}{v_*}}(1-\varepsilon); \tag{1.7}$$

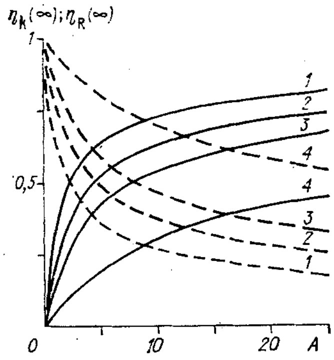

Fig. 1

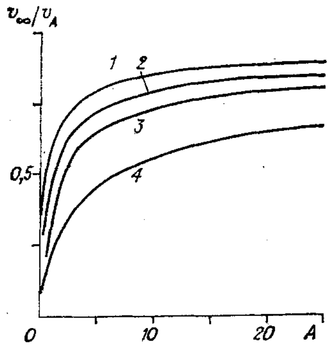

Fig. 2

$$\eta_R = \frac{1-\varepsilon}{1+\frac{v}{v_*}}; \tag{1.8}$$

$$v = v_*\left(\sqrt{\frac{1}{4}+\frac{2W_0}{mv_*^2}(1-\varepsilon)}-\frac{1}{2}\right). \tag{1.9}$$

If it is assumed that by the end of acceleration $\varepsilon \to 0$, the expressions (1.7)-(1.9) permit evaluating the maximum values of $\eta_k(\infty)$, $\eta_R(\infty)$, v_∞. For $R_0 = 0$ $v_* = 0$ and the maximum v_∞, as expected for a nondissipative system, equals

$$v_A = \sqrt{\frac{2W_0}{m}}. \tag{1.10}$$

Substituting (1.9) into (1.7) and (1.8) and normalizing (1.9) to (1.10) we find

$$\eta_k(\infty) = 1-\frac{1}{\sqrt{\frac{1}{4}+A}+\frac{1}{2}}; \tag{1.11}$$

$$\eta_R(\infty) = \frac{1}{\sqrt{\frac{1}{4}+A}+\frac{1}{2}}; \tag{1.12}$$

$$\frac{v_\infty}{v_A} = \frac{\sqrt{A}}{\sqrt{\frac{1}{4}+A}+\frac{1}{2}}; \tag{1.13}$$

$$A = \frac{2W_0}{mv_*^2}. \tag{1.14}$$

It follows from expressions (1.11)-(1.13) that $\eta_k(\infty)$, $\eta_R(\infty)$, and v_∞/v_A depend only on one dimensionless parameter A, formed from the initial determining parameters of the system W_0, m, R_0, and λ; A can be regarded as a similarity criterion for describing the operation of rail guns.

The dimensionless parameters v_* and A are analogous to the characteristic numbers employed in magnetohydrodynamics: the Alfvén velocity and Alfvén's number, the only difference being that v_* characterizes the velocity at which the kinetic energy of the body equals the ohmic losses in the circuit, while the parameter A equals the ratio of the initial energy in the source to the conventional kinetic energy of the body moving with the velocity v_*.

In [1, 2, 4-6], where the operation of rail guns to which power is supplied by a capacitor bank is analyzed, the parameter $q = \lambda^2 C_0{}^2 U_0{}^2/2mL_0$ introduced by L. A. Artsimovich and his colleagues [1] is widely used (λ is the inductance per unit length of the rail gun, C_0 is the capacitance of the capacitor bank, U_0 is the voltage on the bank, m is the mass of the body, and L_0 is the starting inductance of the circuit). Comparison of A and q shows that the parameter A is more general, is more convenient physically, and takes into account the active resistance of the circuit. The relationship between A and q can be represented in the form $A = q\rho_0{}^2/8R_0{}^2$ ($\rho = \sqrt{L_0/C_0}$ is the wave impedance).

2. Simple analytic relations between E_k and E_R and expressions for η_k, η_R, and v can be obtained for the case when the resistance of the circuit changes during the operation of the rail run. Setting

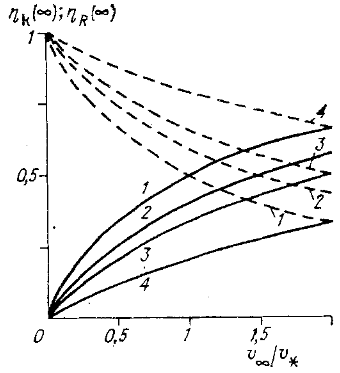

Fig. 3

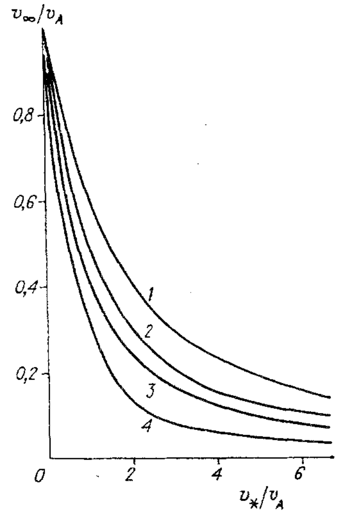

Fig. 4

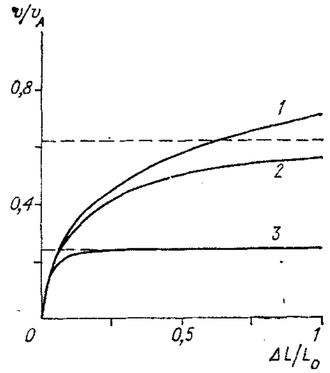

Fig. 5

$$R = R_0 + \alpha v/v_\infty,\ \alpha = \text{const}, \tag{2.1}$$

and following the arguments presented above in the analysis of the energy characteristics for $R = R_0 =$ const. we have

$$\frac{E_k}{E_R} = \frac{\frac{v}{v_*}}{1 + \frac{2\alpha}{\lambda v_\infty}\frac{v}{v_*}}; \tag{2.2}$$

$$\eta_k = \frac{\frac{v}{v_*}(1-\varepsilon)}{1 + \frac{v}{v_*}\left(1 + \frac{2\alpha}{\lambda v_\infty}\right)}; \tag{2.3}$$

$$\eta_R = \frac{\left(1 + \frac{2\alpha}{\lambda v_\infty}\frac{v}{v_*}\right)(1-\varepsilon)}{1 + \frac{v}{v_*}\left(1 + \frac{2\alpha}{\lambda v_\infty}\right)}; \tag{2.4}$$

$$\frac{v}{v_*} = \frac{\sqrt{\frac{1}{4} + A(1-\varepsilon)\left(1 + \frac{2\alpha}{\lambda v_\infty}\right)} - \frac{1}{2}}{1 + \frac{2\alpha}{\lambda v_\infty}}. \tag{2.5}$$

Here v_* and A are determined, as before, by the expressions (1.6) and (1.14); v_∞ is the velocity at which acceleration terminates. Substituting (2.5) into (2.3) and (2.4) we obtain the analytic dependences of η_k, η_R and v on ε, A, α/R_0, v_*/v_∞, whose maximum values for $\varepsilon = 0$ equal

$$\eta_k(\infty) = 1 - \frac{1}{\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{A}{\left(1 + \frac{\alpha}{2R_0}\right)^2}}}; \tag{2.6}$$

$$\eta_R(\infty) = \frac{1}{\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{A}{\left(1 + \frac{\alpha}{2R_0}\right)^2}}}; \tag{2.7}$$

$$\frac{v_\infty}{v_A} = \frac{\sqrt{A}}{\left(1 + \frac{\alpha}{2R_0}\right)\left(\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{A}{\left(1 + \frac{\alpha}{2R_0}\right)^2}}\right)}. \tag{2.8}$$

The parameter α/R_0 characterizes the increment to the resistance of the circuit during the operation of the rail gun. For $\alpha = 0$ the expressions (2.2)-(2.8) are identical, as expected, to (1.11)-(1.13). The dependences of $\eta_k(\infty)$, $\eta_R(\infty)$ (solid and broken lines) and of v_∞/v_A on A are shown in Figs. 1 and 2, the dependences on v_∞/v_A are shown in Fig. 3, and the dependence of v_∞/v_A on v_*/v_A is shown in Fig. 4. Curves 1-4 in Figs. 1-4 were constructed for $\alpha/R_0 = 0$, 1, 2, and 6, respectively.

The expressions (1.5)-(1.13) and (2.1)-(2.8) permit analyzing the energy ratios for a rail gun in the general case. The introduction of the characteristic velocity v_* makes it possible to evaluate the velocity of the body and the efficiency with which the energy of the source is converted into the kinetic energy of the body in terms of the starting parameters of the system W_0, m, R_0, and λ, known initially, and the increment to the resistance of the circuit at the end of acceleration of the body. We note that in our analysis the form of the source of electromagnetic energy was not specified; W_0 can be the energy stored both in a capacitor bank, in inductive storage, or in some other source. The analysis for a given type of source can be continued by substituting a specific expression for W_0 into the formulas obtained. In particular, when W_0 is the energy stored in inductive storage, and $R = \alpha v/v_\infty$, the analytic dependences of the velocity of the body, η_k, and η_R on the increment to the inductance $\Delta L - \lambda x$ (x is the distance traversed by the body) can be derived. Figure 5 illustrates the dependence of v/v_A on $\Delta L/L_0$ (curves 1-3 correspond to $\alpha/\lambda v_A = 0$, 0.5, and 2).

LITERATURE CITED

1. L. A. Artsimovich, S. Yu. Luk'yanov, et al., "Electrodynamic acceleeration of plasmoids," Zh. Eksp. Teor. Fiz., 33, No. 3 (1957).
2. J. G Linhart, "A simplified analysis of the dynamics of plasma guns," Nuclear Fusion, 1, No. 2 (1961).
3. V N. Bondletov, "Determination of the electromagnetic forces, their work, and electromechanical efficiency in circuits with currents," Elektrichestvo, No. 1 (1966).
4. L. V. Dubovoi and Yu. A. Berezin, "Choice of optimal parameters of a plasma gun," Zh. Tekh. Fiz., 34, No. 10 (1964).
5. A. B. Novgorodtsev and G. A. Shneerson, "Energy relations in an oscillatory circuit, employed for accelerating conductors by electromagnetic forces," Izv. Akad. Nauk SSSR, Énergetika i Transport, No. 2 (1970).
6. G. A. Shneerson, Fields and Transient Processes in Ultrastrong Current Apparatus [in Russian], Énergoizdat, Leningrad (1981).
7. T. J. Burgess and M. Cowan, "Multistage induction mass accelerator," IEEE Trans., MAG-20, No. 2 (1984).
8. D. R. Sadelin, "Efficiency equations of the rail gun," ibid.
9. G. A. Clark and A. J. Bedford, "Performance results of a small-caliber electromagnetic launcher," ibid.